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The Half-Cycle Correction Explained: Two Alternative Pedagogical Approaches

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While teaching both an introductory and advanced course in decision analysis at the University of Toronto over the past 15 years, we have found that a perennial problem for our students and ourselves is the explanation of the half-cycle correction employed in Markov models. The half-cycle correction is very often used to compensate for the fact that in discrete, computer-based approximations of the ideal Markov process is shown. Students are able to clearly see that the cumulative incremental utility in the discrete case underestimates the desired quantity. Likewise, they find the concept of shifting the ideal curve to the right by one-half cycle to reduce the latter discrepancy to be intuitive. However, students often find the approximate equivalence of shifting the ideal state membership curve and adding a half-cycle’s worth of incremental utility to the total for the state at the beginning of a discrete Markov process to be a difficult cognitive leap. This article describes 2 pedagogical devices, algebraic and intuitive/visual approaches, that may assist the instructor of Markov theory to convey the latter concept. Elements of adult learning theory are discussed, which may help the instructor to choose which approach to employ. Implementation of the half-cycle correction in commonly used decision-analytic software is also discussed. Key words: Markov chains; models; educational; statistical. (Med Decis Making 2008;28:706–712)

THE USUAL PEDAGOGICAL APPROACH TO THE HALF-CYCLE CORRECTION

The usual pedagogical approach is to present students with a figure displaying a stepwise, computed, state membership graph representing the output from a discrete Markov process in relation to an ideal, continuous membership function. The latter is represented as a smoothly declining, monotonic curve (Figure 1). The effect of accounting for state membership at the beginning or end of each cycle is shown with the consequent over- and underestimation of the true state membership, respectively. The effect of shifting the computed, stepwise graph one half-cycle to the right is then shown (Figure 1C) with the desired effect of reducing the deviation of the computed cumulative state membership from the true value. By visual inspection, students can easily appreciate that shifting the ideal curve to the right by one-half cycle is equivalent to counting state membership in the middle of each cycle.

When it comes time for students to build Markov models using commonly available decision analytic software, they find that state membership must be counted either at the beginning or the end of each cycle rather than in the middle. Students are taught that, if they use a software application where state...
membership is counted at the end of each cycle, they should add one-half of a cycle’s worth of incremental utility to the cumulative total for each state in the model—that is, the “half-cycle correction.” They are taught that the half-cycle correction is roughly equal to the effect of shifting the computed state membership graph as described above. In our experience, students often have trouble understanding the latter concept: that is, why adding a half-cycle of incremental utility to a Markov process where state membership is counted at the end of each cycle approximates shifting the curve so that, in effect, membership is counted in the middle.

We offer 2 alternative approaches to explaining this frequently misunderstood concept. For the purpose of exposition, we assume that state membership is counted at the end of each cycle. Both methods seek to demonstrate to students that the difference in the area underneath the ideal, smooth membership function and the stepwise, computed membership graph is approximately equal to the area encompassed by a rectangle that is 1 unit tall and half of a cycle wide. The addition of that rectangle to the beginning of the stepwise, computed membership graph achieves the curve shift shown in Figure 1C.

**AN ALGEBRAIC EXPLANATION**

Let us assume that there is a particular Markov state where the true fraction of the hypothetical cohort remaining in that state declines according to a smooth, declining monotonic function. Furthermore, assume that a discrete Markov process is used to approximate the latter function and that it accounts for membership at the end of each cycle. Thus, within the discrete approximation, the incremental utility generated for each cycle by the hypothetical individuals residing in that state can be represented by the area of a rectangle whose base is equal to the cycle-length and whose height is equal to the vertical displacement on the y-axis of a point on the true state membership curve at the end of the cycle (Figure 2).
The area enclosed by this rectangle will underestimate the true membership in the state by an amount that is equal to the area in the region bounded by the true state membership curve and the top of the rectangle. Although the region above the rectangle is not strictly triangular (due to the curve of the true membership graph), one can approximate this area with a triangle provided that the cycle-length is not too long in relation to the termination time of the Markov process. Close inspection of these triangular areas will reveal that they sum to an amount equivalent to a half-cycle. However, this fact can be formally proven to students.

The base of each triangle is equal to the cycle-length “B,” and the height of the $i$th triangle is equal to the difference in height between the $(i-1)$th rectangle and the $i$th rectangle (Figure 2):

$$A_i \approx \frac{1}{2}B(H_{i-1} - H_i).$$

Thus, the total area of the triangles is

$$A = \sum A_i \approx \frac{1}{2}B(1 - H_1) + \frac{1}{2}B(H_2 - H_1) + \frac{1}{2}B(H_3 - H_2) + \ldots + \frac{1}{2}B(H_{n-1} - H_n).$$

So that

$$A \approx \frac{1}{2}B(1 - H_1) + (H_2 - H_1) + (H_3 - H_2) + \ldots + (H_{n-1} - H_n).$$

And canceling terms yields

$$A \approx \frac{1}{2}B[1 - H_n].$$

Now since $H_n \approx 0$, provided that “n” is large enough, one finds that

$$A \approx \frac{1}{2}B \cdot [1].$$

In other words, this simple and straightforward proof demonstrates that the discrepancy between the computed, cumulative state membership from the Markov process (the rectangles)—when membership is counted at the end of a cycle—and the true cumulative area is approximately equal the area of a rectangle with a height of one and a base of one-half of a cycle-length (i.e., a half-cycle). Thus, students will hopefully see the necessity of adding this half-cycle correction to the total cumulative membership for the state produced by the discrete Markov process. Furthermore, the addition of the half-cycle rectangle at the beginning of the discrete Markov function is equivalent to the half-cycle curve shift seen in Figure 1C.

**WHEN THE MARKOV PROCESS TERMINATES EARLY**

The algebraic approach may be useful in demonstrating to students why the half-cycle correction must be adjusted if the Markov process is terminated early—that is, when a substantial fraction of the cohort remains in the state. If the process is terminated after 8 cycles, then the half-cycle correction would be too large by an amount approximately equal to the area of the sum of the ninth to the last triangles. This latter area is

$$A_{9 \to n} \approx \frac{1}{2}B[(H_9 - H_8) + (H_9 - H_{10}) + (H_{10} - H_{11}) + \ldots + (H_{n-1} - H_n)].$$

And canceling terms yields

$$A_{9 \to n} \approx \frac{1}{2}B[H_{8} - H_n].$$

Now, again, since $H_n \sim 0$, one finds that

$$A_{9 \to n} \approx \frac{1}{2}B[H_8].$$

In other words, if a half-cycle had been added to the cumulative, discrete state membership function and the process is terminated early, a quantity must be subtracted from the cumulative total. This amount is equivalent to the area of a rectangle with a base equal to one half of a cycle-length and a height equal to the fraction of the cohort remaining in the state at the end of the last cycle (Figure 3).

**AN INTUITIVE ALTERNATIVE**

The algebraic explanation may not satisfy some students. We therefore offer an intuitive/visual explanation as an alternative (Figure 4). Assuming that loss of members from a health state occurs continuously and proportionally throughout a cycle, then the true state membership can be described by a smooth, declining curve (Figure 4a). When we perform discrete Markov analyses, we approximate that behavior by assuming that patients exit the state at fixed time intervals (cycles). We usually account for these losses at the end of the cycle. The discrete process is represented by the rectangles underneath the smooth curve (Figure 4a).

The sum of the areas of the rectangles is equal to the estimate of the total number of cycles accrued by the members residing in the health state. But it can be appreciated that the sum of the rectangles...
underestimates the true number of cycles accrued by an amount equal to the areas between the tops of the rectangles and the smooth curve (Figure 4a). We will refer to the latter area as the area of interest. We would like to add the area of interest to the summed areas of the rectangles to obtain a more accurate value for the total number of cycles experienced by the cohort in the health state.

To improve our estimate, we first create additional rectangular areas indicated by the dotted lines in Figure 4b. We then disregard the original rectangles for the time being and focus only on the new ones (Figure 4c). For simplicity, we replace the smooth curve with straight line segments, which divides the new rectangles diagonally in half (Figure 4d). The area of interest is now represented by the lower left triangle within each rectangle. It is easy to appreciate that those lower left triangular regions represent half of the area of the new rectangles.

In the next step, we imagine sliding the new rectangles to the left so that they line up against the y-axis of the graph. It is easy to see, by visual inspection, that the rectangles stack neatly between 0 and 1 and that they are all 1 cycle wide (Figure 4e). Since the areas of interest encompass half of each rectangle, and the rectangles stack neatly between 0 and 1, one can see visually that the total area of interest is equivalent to a rectangle one-half cycle wide and whose height extends from 0 to 1 (Figure 4f).

Finally, we add back the original rectangles (Figure 4f) and include our new half rectangle. One can see that the effect of this is to shift our discrete process to the right by half a cycle. Now the smooth curve runs, more or less, through the middle of the original rectangles so that the fit is better. Thus, by adding an additional half-cycle to the discrete estimate (i.e., by making a half-cycle correction), we improve the accuracy of the discrete Markov process.

**APPLYING ADULT LEARNING THEORY TO THE PROBLEM OF EXPLAINING THE HALF-CYCLE CORRECTION**

Determining whether to employ an algebraic or an intuitive/visual explanation of the half-cycle correction concept may depend on the learning preference of the individual student. A large theoretical and empiric literature exists that explores the issue of learning preferences. For example, the Dunn and Dunn learning model postulates that the educator should match learning preferences to maximize the participant’s learning. Educational researchers have demonstrated that matching perceptual preferences improves immediate recall (short-term memory), delayed recall (long-term memory), and attitudes in adult learners. Researchers have also demonstrated improved attitudes toward the subject matter when perceptual preferences were matched. These findings suggested that matching perceptual preferences translated into a more satisfactory educational experience.

The terms *analytic* and *global* have often been used to describe cognitive or information processing styles. Based on information processing theory, analytic students learn more easily through a step-by-step sequential process, which builds toward conceptual understanding. This learning process mirrors traditional teaching methods. Conversely, students with a global learning style are able to comprehend more easily when they understand the overall concept first, and then concentrate on the details later. For global students, information is best presented with many examples, which are based on actual practice (i.e., case-based).

When confronted with a mathematical construct, for example, the analytic student would learn best via a stepwise algebraic proof using symbolic notation. However, the global student would achieve understanding most easily if the overall purpose of the construct were explained first and then a set of examples using actual numbers were presented. If
Figure 4  An intuitive/visual explanation of the half-cycle correction. A, The underestimation of a discrete Markov process with state membership counted at the end of each cycle compared to a true continuous process is represented by the area between the tops of the rectangles and the smooth curve. B, Create new rectangular areas. C, Temporarily disregard the original discrete process rectangles. D, Replace the smooth curve with straight line segments. E, Slide the new rectangular areas so that they line up against the y-axis. F, The triangular areas can be seen to be equal to a rectangle one-half cycle wide extending from 0 to 1. G, Add back the original rectangles; the approximation is better.
the Dunn and Dunn conjecture and if information processing theory are both valid, then the algebraic explanation of the half-cycle correction should be employed for analytic learners whereas the intuitive approach should be used for global students.

**HALF-CYCLE CORRECTION ISSUES ASSOCIATED WITH COMMON DECISION-ANALYTIC SOFTWARE**

Students enrolled in our decision analysis courses most often use either the TreeAge\textsuperscript{1} or Decision Maker programs for constructing Markov-based decision models. It is important that they understand the differences between these 2 programs with respect to the half-cycle correction. Decision Maker accounts for state membership at the end of each cycle so that neither of the pedagogical devices mentioned above needs to be altered.\textsuperscript{14} Each Markov state has 3 special variables associated with it: “m.uINIT” represents a quantity of incremental utility to be added at the beginning of the discrete Markov process; “m.uINCR” represents the incremental utility of subsequent cycles; and “m.uTAIL” represents the incremental utility to be added when the Markov process terminates for the portion of the cohort remaining in the state. If “X” represents the incremental utility associated with the state, then the half-cycle correction in Decision Maker is achieved by setting m.uINIT, m.uINCR, and m.uTAIL to 0.5*X, X, and −0.5*X, respectively.

In contrast, TreeAge accounts for state membership at the beginning of each cycle rather than at the end and therefore overestimates true state membership.\textsuperscript{15} Thus, a half-cycle must be subtracted from the cumulative total for a given health state. If the process terminates early, then a half-cycle must be added back for the proportion of the cohort remaining in the state. A potentially confusing aspect of TreeAge is that the users’ manual indicates that transitions occur between states at the end of each cycle. However, students should understand that the important criterion with respect to the half-cycle correction is not when the transitions occur but, rather, when state membership is counted.

The very first cycle in a Markov process constructed with the TreeAge program is counted as cycle 0. Within that initial cycle, the half-cycle penalty is assessed by adding only half of the incremental utility to the cumulative total that would normally accrue in a typical cycle. If no half-cycle correction is to be assessed, then the value for cycle zero is the same as for the subsequent cycles. If the process terminates early, then a half-cycle needs to be added back for the proportion of the cohort remaining in that state. TreeAge allows the users to specify the initial utility for cycle 0, the incremental utilities for all subsequent cycles, and the terminal utility in case the Markov process stops early for each state. One might think that the initial and terminal utilities should have opposite signs since one is a half-cycle subtraction and the other is an addition, respectively. However, this is not the case. The terminal utility should have a positive sign because it is a half-cycle addition, whereas, for the initial utility, the subtraction is implied: the initial utility is only half of the value that would have been accrued if the half-cycle correction had not been assessed.

TreeAge refers to incremental utility as incremental rewards of which there are 3 for each Markov state: “Init” refers to the reward for cycle 0; “Incr” is the reward for all subsequent cycles; and “Final” is the reward to be assigned if the process terminates early. If “X” represents the incremental reward associated with the state, then the half-cycle correction in TreeAge is achieved by setting Init, Incr, and Final to 0.5*X, X, and 0.5*X, respectively.

Students who use other software for conducting Markov analyses should read the accompanying documentation carefully and pay close attention to the order in which calculations are performed during each cycle. If state transitions occur before state membership is counted, then half-cycle corrections should be implemented as it is for Decision Maker. Otherwise, a TreeAge-like approach is required and they should determine how their chosen program handles the initial cycle.

**CONCLUSION**

We hope that these pedagogical devices and approaches will prove useful to instructors of Markov theory as they explain the rough equivalence between the half-cycle correction and the state membership curve shift to their students. Empirical studies of the cognitive learning processes and preferences of students of Markov-based decision analyses is an area for future research.

**REFERENCES**